Digital Hydraulic Actuator Control Using an Electro-Hydraulic Poppet Valve (EHPV™)

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ABSTRACT
This paper addresses the design of a digital control system for a hydraulic circuit consisting of a novel Electro-Hydraulic Poppet Valve (EHPV™), a fix displacement pump, an actuator, and a tank. This paper first investigates the mathematical model of the EHPV™, which results in a 5th order system. Under the assumption of incompressible turbulent fluid flow, it is shown that the EHPV™ can be modeled as a 4th order system. The model of the EHPV™ is then compounded with the model of the hydraulic circuit. The resulting mathematical model for which the control design is targeted is found to be 6th order. For such high order system, the control design was implemented using modern digital state space techniques, which include an integral controller and a deadbeat full order predictive observer. Hence, the pole placement problem is addressed for a 7th order system. The control design was then tested through simulations that include disturbances and parameter variations to study its robustness.

1. Introduction
The current trend in construction machinery is to use electrically controlled valves (solenoid valves) instead of manually operated hydraulic valves. One of the benefits is that these solenoid valves need not be located in the operator cab. In addition, the employment of these electrically driven valves facilitates computerized control of various machine functions. The Electro-Hydraulic Poppet Valves (EHPV™), a kind of solenoid valves, are used for flow control in hydraulic machinery. The flow control through the valve is achieved by changing the valve restriction coefficient via a poppet type orifice with pressure compensation. The integrated electronics makes practical advanced control algorithms to further extend the valve capabilities in new ways and its application. This paper will explore the use of such valve to control the motion of an actuator piston that is hydraulically driven. Mainly, the purpose is to design a digital controller to displace the actuator piston by 0.2 meters under 0.5 seconds with low overshoot. The design will be done by using modern control design techniques.

2. Valve Operation
The EHPV™ is used to proportionally control flow rate through the valve. To accomplish this, the high pressure flow (inflow) is connected to the control pressure chamber through a small channel inside the main poppet. Then, applying a given input current causes the pilot poppet to displace and hence open an aperture through which fluid is bled off to the low pressure connection (output connection). Such action causes a pressure imbalance that pushes the main poppet away from its seat thus enabling a direct passage between the inlet and outlet connection. A representative flow diagram is given in Figure 2.1. In this figure, the high pressure flow is denoted by $Q_a$. This flow is bifurcated into $Q_1$ that goes to the control pressure chamber and $Q_2$, which directly goes to the outlet connection (in this schematic, the channel inside the main poppet through which fluid is sent to the control chamber is represented outside of the main poppet for clarification purposes). $Q_p$ is the pilot flow that mixes with $Q_1$ to become the output flow $Q_b$. It is important to mention that the position of the pilot and the main poppet shown in this schematic correspond to a given equilibrium state for which there is a nonzero input.

3. Mathematical Model
The EHPV™ mathematical model is derived using the following equations under the assumption that the input dynamics (i.e. the dynamics of the electric circuit through which current flows in and out of the solenoid) are much faster than the valve dynamics. Equation 3.1 shows that the direct flow $Q_1$ is proportional to the displacement of the main poppet $x_m$ and the square root of the inlet-outlet pressure differential.
Similarly, Equation 3.2 shows that the branch flow $Q_2$ is proportional to the displacement of the main poppet $x_m$ ($Q_2$ is a constant value) and the square root of the inlet-control pressure differential. The pilot flow, however, is proportional to the relative position between the pilot and the main poppet while still being proportional to the square root of the control-output pressure differential.

$$Q_i = R_m(x_m)^2 \sqrt{P_a - P_b}$$  \hspace{1cm} (3.1)

$$Q_p = R_p(x_m^2)^2 \sqrt{P_a - P_p}$$  \hspace{1cm} (3.2)

$$Q_p = R_p(x_m^2)^2 \sqrt{P_a - P_p}$$  \hspace{1cm} (3.3)

$$Q_p = Q_p + Q_1$$  \hspace{1cm} (3.4)

The time rate of change of the control pressure is calculated with the aid of Equation 3.5 under the assumption that the control pressure chamber volume is negligible compared to $Q_2$ and $Q_p$ (turbulent flows).

$$\frac{dP}{dt} = \frac{\beta}{a_{m,1}(x_o - x_m)} (Q_2 - Q_p)$$  \hspace{1cm} (3.5)

The dynamics of the pilot are given by Equation 3.6. This equation is obtained from a given equilibrium state of the pilot, different from the closing state, and indicates that flow forces that depend on changes measured about the equilibrium values. If the input and known pressures are denotes as:

$$u_v = \bar{u_v} + \delta u_v$$

$$P_a = \bar{P_a} + \delta P_a$$

$$P_p = \bar{P_p} + \delta P_p$$

then the valve mathematical model is given by Equation 3.10.

$$X = f_1(X) + B_1 \delta u_v$$  \hspace{1cm} (3.10.a)

The delta symbol $\delta$ indicates quantities measured about the equilibrium state of the main poppet, different from the closing state, and indicates that flow forces that depend on changes measured about the equilibrium values.

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} \delta \dot{X}_1 \\ \delta \dot{X}_2 \\ \delta \dot{X}_3 \\ \delta \dot{X}_4 \\ \delta \dot{X}_5 \end{bmatrix}$$  \hspace{1cm} (3.10.b)

$$f_1(X) = \begin{bmatrix} -\frac{k_p}{m_p} X_1 - \frac{b_p}{m_p} X_2 + \frac{a_{m,1}}{m_p} X_3 + \frac{a_{m,2}}{m_p} \delta P_a + \frac{a_{m,3}}{m_p} \delta P_p \\ \sigma_- \end{bmatrix}$$  \hspace{1cm} (3.10.c)

$$\sigma = R_p(D_x - \bar{x}_m - \bar{X}_m) \sqrt{P_a + \delta P_a - \bar{P}_a - X_1}$$

$$- R_p(D_x - \bar{x}_m - \bar{X}_m) \sqrt{P_p + \delta P_p - \bar{P}_p - X_1}$$

$$Q_p = Y_o = R_p(\bar{x}_m + \bar{X}_m) \sqrt{P_p + \delta P_p - \bar{P}_p - \bar{P}_r}$$  \hspace{1cm} (3.10.d)

where $Y_o$ represents the equilibrium value while the symbol $\delta$ indicates changes measured about the equilibrium values. If a nearly incompressible fluid is used (such as hydraulic oil), then Equation 3.5 can be modified:

$$a_{m,1}(x_o - x_m) P_p \approx 0$$  \hspace{1cm} (3.11)

which yields the fact that $Q_o^* = Q_p^*$. Therefore, from this relationship, an algebraic expression can be found to relate $\delta P_p$ to the other states. Substituting this result into Equation 3.10.b through 3.10.d yields a fourth order system:
\[ \dot{X} = f(X) + B \delta u \] (3.12.a)

\[
X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} \dot{x}_m \\ \dot{\delta}_w \\ \dot{\delta}_p \end{bmatrix} \quad (3.12.b)
\]

\[
f(X) = \left[ \begin{array}{c}
\frac{k_w}{m_w} X_i - \frac{h_u}{m_u} X_i - \frac{a_w}{m_u} \delta_w + \frac{a_w}{m_u} (\delta_p - \delta_p^c) \\
- \frac{k_w}{m_p} X_i - \frac{h_u}{m_p} X_i + \frac{a_p}{m_p} \delta_p \\
\end{array} \right] \quad (3.12.c)
\]

\[
B = \begin{bmatrix} 0 \\ 0 \\ \frac{K}{m_p} \\
\end{bmatrix} \quad (3.12.d)
\]

\[
Q_s = Q_m = R_a (\tau_a + X_1) \left[ \frac{P_T + \delta P_T - \delta p^c}{R} \right] \quad (3.10.e)
\]

The dynamics of the piston in the actuator are accounted in the following equation, in which \( \delta \) is the displacement of the piston from its equilibrium position (likewise, nonzero equilibrium position). It can be observed that the change in the lower cavity pressure \( \delta p_T \) drives the piston.

\[
m_p \delta \ddot{y} + b_p \dot{\delta} \dot{y} + k_p \delta \dot{y} = a_p \delta P_T \quad (4.1)
\]

Using the same assumption applied in Equations 3.5 and 3.11, it can be stated that the inflow to the lower cavity of the actuator is the same outflow to tank, as given in Equation 4.2.

\[
0 = Q_s - Q_T \quad (4.2)
\]

From this equation, a relationship can be established between \( \delta P_T \) and the tank pressure \( P_T \) (constant) as shown next, in which \( Q_s \) is substituted from Equation 3.10.e, and \( R_T \) is the orifice coefficient for \( Q_s \).

\[
\delta P_T = \left( \frac{Q_s}{R_T} \right)^2 + \left( \frac{P_T - P_T^c}{R_T} \right) \quad (4.3)
\]

Substituting Equation 4.3 into Equation 4.1 and adding the two new states to the state space representation of the EHPV™ given by Equation 3.12 yields the complete mathematical model of the system to be controlled. This complete system model is given in Equation 4.4, in which \( \delta P_T \) is substituted from Equation 4.3.

\[
\dot{X} = F(X) + R \delta u \quad (4.4.a)
\]

\[
\dot{\delta y} = CX \quad (4.4.b)
\]

\[
X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} \delta x_m \\ \delta \dot{x}_m \\ \delta x_p \\ \delta \dot{x}_p \\ \delta y \\ \delta \dot{y} \end{bmatrix} \quad (4.4.b)
\]

\[
F(X) = \left[ \begin{array}{c}
\frac{k_w}{m_w} X_i - \frac{h_u}{m_u} X_i - \frac{a_w}{m_u} \delta_w + \frac{a_w}{m_u} (\delta_p - \delta_p^c) \\
- \frac{k_w}{m_p} X_i - \frac{h_u}{m_p} X_i + \frac{a_p}{m_p} \delta_p \\
\end{array} \right] \quad (4.4.c)
\]

**4. Complete Actuator-EHPV™ Mathematical Model**

The EHPV™ is used to control the displacement of an actuator piston as depicted in Figure 4.1.

![Figure 4.1. Complete System.](image)

Figure 4.1 shows how hydraulic fluid is received from a fixed displacement pump [a] at the EHPV™ [b]. When an input current is applied, the solenoid [b] enables hydraulic fluid to go through the EHPV™ [b] and then through a check valve [c] to the lower cavity [d] of the actuator [d]. The incoming fluid displaces the actuator piston [d] at the same time that the actuator spring [d] is compressed. After it passes the actuator [d], hydraulic fluid is dumped into a tank [e] from which it is pumped again to close the cycle.
5. Jacobian Linearization

The jacobian linearization about the equilibrium state was obtained with the aid of the MATLAB command Jacobian. The resulting 6th order linearized system with the correspondent values substituted is:

\[
\begin{align*}
    \dot{X} &= A_L X + B_L \delta u, \\
    \delta y &= C_L X
\end{align*}
\] (5.1.a)

where the equilibrium state is given by:

\[
X = \begin{bmatrix}
    0.0010 \text{m} \\
    0 \text{m/s} \\
    0.0019 \text{m} \\
    0 \text{m/s} \\
    0.3000 \text{m} \\
    0 \text{m/s}
\end{bmatrix}
\] (5.2)

6. Discretization

The state space representation of Equation 5.1 is then discretized using a zero-order-hold (ZOH). The sampling rate was selected to be 1kHz, which yields a sampling period \( T_s \) of 0.001 seconds. The resulting discretized state space representation is given in Equation 6.1.

Before any design is attempted, it is necessary to check for controllability and observability if one wishes to implement a control scheme such as that in Figure 8.1. Consequently, the rank of the controllability matrix \( M \) and the observability matrix \( Q \) of the discrete system given in Equation 6.1 were first checked. As a result, both matrices were full rank.

The next step in the design is the selection of the state feedback gain \( K \) and the integral gain \( K_I \). In order to accomplish this task, a new state space representation is developed to include the integrator state.
\[
\eta(k+1) = (\hat{G} - \hat{H} \hat{K}) \eta(k) + W \quad (8.1)
\]

\[
\hat{G} = \begin{bmatrix} G & 0 \\ C & 1 \end{bmatrix}, \quad \hat{H} = \begin{bmatrix} H' & 0 \end{bmatrix}, \quad \hat{K} = \begin{bmatrix} K & K_I \end{bmatrix} \quad (8.2)
\]

In order to find out the closed loop pole locations for the state feedback control law, the DLQR (Discrete Linear Quadratic Regulator) function in MATLAB was first employed for the system excluding the integrator. A pole at zero was added to the previously found poles, and they were used to compute the gains \( K \) and \( K_I \) using the Ackerman formula given next.

\[
\begin{bmatrix} K & K_I \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \hat{M}^{-1} \Phi(\hat{G}) \quad (8.3)
\]

In this equation, the controllability matrix was calculated using the new \( \hat{G} \) and \( \hat{H} \) matrices given in Equation 8.2. In addition, \( \Phi \) is the desired characteristic equation. The calculated gains are presented in Equation (8.4).

\[
K = \begin{bmatrix} 2671 & 125 & 8861 & 375 & 893 & 9.14 \end{bmatrix}, \quad K_I = 7.85
\quad (8.4)
\]

It is intended that the state feedback gain \( K \) acts upon the values of the states. However, this is possible only if the values of the states are known either from measurements or from a given estimation. Even though the EHPV™ design allows control pressure measurements, these measurements can only be substituted in the reduced order system (the 4th order model) since this control pressure is no longer a state.

On the other hand, one could have measurements of the position, velocity, and acceleration of the piston. Nevertheless, for simplicity a deadbeat full order predictive observer was employed to estimate the states. The calculation of the observer gain \( L \) was accomplished using Equation 8.5. Furthermore, Equation 8.6 gives the calculated observer gain.

\[
L = G(s)Q^{-1} \quad (8.5)
\]

\[
L = \begin{bmatrix} -369 & -27957 & 175 & 21110 & 6 & 8092 \end{bmatrix} \quad (8.6)
\]

9. Simulations
The closed-loop system given in Figure 8.1 was implemented in SIMULINK in order to perform the simulations. Furthermore, the piston displacement response from the equilibrium position with no disturbances is presented in Figure 9.1, in which it can be seen that the steady state error is zero. Figure 9.2 shows a close-up view of Figure 9.1 to illustrate that the closed-loop response meets the rest of the specifications. It can be seen in this graph that there is no overshoot and that the settling time is less than 0.5 seconds, both of which comply with the requirements.

![Figure 9.1 Piston Displacement from Equilibrium Position.](image1)

![Figure 9.2 Close-up View of Previous figure.](image2)

The input current to the EHPV™ about the equilibrium current is presented in Figure 9.3. As it can be noticed, the input need does not deviate much from the equilibrium value.

![Figure 9.3 Input Current Applied to the EHPV™.](image3)
Usually, uncertainties in the model appear as disturbances. Furthermore, disturbances can arise from many other things such as the piston encountering an obstacle in its path, or poppet opening effects. The piston displacement response that includes these last two types of disturbances is presented next. It can be concluded that the system recovers and the piston reaches the desired position.

11. Conclusions
The mathematical model of the EHPV™ was examined and it was concluded that this type of valve can be represented as a 4th order system. This resulting model was then incorporated into the hydraulic circuit, which yielded a 6th order system. A control system was then designed to control the displacement of an actuator piston from equilibrium using an integral controller and a deadbeat observer. It was found from simulations that the design satisfactorily changed the behavior of the system to one that complies with the given specifications. Furthermore, the input behavior was assessed and it was found to be reasonable. Moreover, the system responded well to disturbances and it was shown that the closed-loop system response remains stable for 20% variations in the value of the pilot and main poppet masses. Finally, it was mentioned that the control design is more sensitive to parameter variations in the piston values.

12. Nomenclature

- $a_{up}$: upper area of main poppet $m^2$
- $a_{lp}$: lower area of main poppet $m^2$
- $a_{p}$: cross sectional area of the pilot $m^2$
- $a_{p}$: cross sectional area of piston $m^2$
- $\beta$: bulk modulus $Pa$
- $b_{p}$: damping coefficient of main poppet $N/(m/s)$
- $b_{up}$: damping coefficient of pilot poppet $N/(m/s)$
- $b_{lp}$: damping coefficient of piston $N/(m/s)$
- $\Delta p$: change in inlet pressure $Pa$($g$)
- $\Delta p_{in}$: change in outlet pressure $Pa$($g$)
- $\Delta p_{in}$: change in control pressure $Pa$($g$)
- $\Delta p_{in}$: displacement of pilot poppet $m$
- $\delta_{p}$: displacement of piston $m$
- $D_1$: feed line hydraulic diameter $m$
- $K_a$: magnetic gain $N/Amps$
- $k_a$: stiffness of main poppet spring $N/m$
- $k_p$: stiffness of pilot poppet $N/m$
- $k_{ps}$: stiffness of piston spring $N/m$
- $M$: controllability matrix
- $m_a$: mass of the main poppet $kg$
- $m_p$: mass of the pilot poppet $kg$
- $m_{ps}$: mass of the main poppet $kg$
- $P_{t}$: tank pressure $Pa$($g$)
- $Q$: observability matrix
- $Q_{o}$: direct input-output flow $m^3/s$
- $Q_{b}$: branch flow from input to control chamber $m^3/s$
- $Q_{i}$: input flow $m^3/s$
- $Q_{o}$: orifice $m^3/s$
- $Q_{p}$: pilot passage flow $m^3/s$
- $Q_{t}$: piston to tank flow $m^3/s$
- $R_e$: orifice coefficient for $Q_{p}$ $[m^2/s]/sqrt(Pa(g))$
- $R_p$: orifice coefficient for $Q_{b}$ $[m^2/s]/sqrt(Pa(g))$
- $R_{np}$: orifice coefficient for $Q_{e}$ $[m^2/s]/sqrt(Pa(g))$
- $R_{ps}$: orifice coefficient for $Q_{ps}$ $[m^2/s]/sqrt(Pa(g))$
- $u_{c}$: input current $Amps$
- $x_{b}$: maximum main poppet stroke $m$

13. References


